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Atoms

Multiple Choice Questions (MCQs)

Q. 1 Taking the Bohr radius as $a_0 = 53$ pm, the radius of Li^{++} ion in its ground state, on the basis of Bohr's model, will be about

(a) 53 pm
(b) 27 pm
(c) 18 pm
(d) 13 pm

K **Thinking Process**

Since, the radii of the orbits increase inversely as atomic number Z i.e.,

$$r \propto \frac{1}{Z}.$$

Ans. *(c)* The atomic number of lithium is 3, therefore, the radius of Li^{++} ion in its ground state, on the basis of Bohr's model, will be about $\frac{1}{3}$ times to that of Bohr radius.

Therefore, the radius of lithium ion is near $\frac{53}{3} \approx 18$ pm.

Q. 2 The binding energy of a H-atom, considering an electron moving around a fixed nuclei (proton), is

$$B = -\frac{me^4}{8n^2\varepsilon_0^2h^2} \quad (m = \text{electron mass})$$

If one decides to work in a frame of reference where the electron is at rest, the proton would be moving around it. By similar arguments, the binding energy would be

$$B = -\frac{Me^4}{8n^2\varepsilon_0^2h^2} \quad (M = \text{proton mass})$$

This last expression is not correct, because

(a) n would not be integral

(b) Bohr-quantisation applies only two electron

(c) the frame in which the electron is at rest is not inertial

(d) the motion of the proton would not be in circular orbits, even approximately.

K **Thinking Process**

The electron revolves uniformly around nucleus have certain centripetal acceleration associated with it.

Ans. ***(c)*** When one decides to work in a frame of reference where the electron is at rest, the given expression is not true as it forms the non- inertial frame of reference.

Q. 3 The simple Bohr model cannot be directly applied to calculate the energy levels of an atom with many electrons. This is because

(a) of the electrons not being subject to a central force

(b) of the electrons colliding with each other

(c) of screening effects

(d) the force between the nucleus and an electron will no longer be given by Coulomb's law

K **Thinking Process**

The electrostatic force of attraction between electron and nucleus is a central force which provide necessary centripetal force for circular motion of electron.

Ans. ***(a)*** The simple Bohr model cannot be directly applied to calculate the energy levels of an atom with many electrons. This is because of the electrons not being subject to a central force.

Q. 4 For the ground state, the electron in the H-atom has an angular momentum $= h$, according to the simple Bohr model. Angular momentum is a vector and hence there will be infinitely many orbits with the vector pointing in all possible directions. In actuality, this is not true,

(a) because Bohr model gives incorrect values of angular momentum

(b) because only one of these would have a minimum energy

(c) angular momentum must be in the direction of spin of electron

(d) because electrons go around only in horizontal orbits

K **Thinking Process**

Bohr's second postulate defines these stable orbits. This postulate states that the electron revolves around the nucleus only in those orbits for which the angular momentum is some integral multiple of $\frac{h}{2\pi}$ *where h is the Planck's constant* $(= 6.6 \times 10^{-34}$ *J-s).*

Ans ***(a)*** In the simple Bohr model, only the magnitude of angular momentum is kept equal to some integral multiple of $\frac{h}{2\pi}$, where, h is Planck's constant and thus, the Bohr model gives incorrect values of angular momentum.

Q. 5 O_2 molecule consists of two oxygen atoms. In the molecule, nuclear force between the nuclei of the two atoms

(a) is not important because nuclear forces are short-ranged
(b) is as important as electrostatic force for binding the two atoms
(c) cancels the repulsive electrostatic force between the nuclei
(d) is not important because oxygen nucleus have equal number of neutrons and protons

K Thinking Process

The nuclear force is much stronger than the Coulomb force acting between charges or the gravitational forces between masses. The nuclear binding force has to dominate over the Coulomb repulsive force between protons inside the nucleus.

This happens only because the nuclear force is much stronger than the Coulomb force. The nuclear force between two nucleons falls rapidly to zero as their distance is more than a few femtometres.

Ans. *(a)* In the molecules, nuclear force between the nuclei of the two atoms is not important because nuclear forces are short-ranged.

Q. 6 Two H atoms in the ground state collide inelastically. The maximum amount by which their combined kinetic energy is reduced is

(a) 10.20 eV (b) 20.40 eV (c) 13.6 eV (d) 27.2 eV

K Thinking Process

The lowest state of the atom, called the ground state, is that of the lowest energy, with the electron revolving in the orbit of smallest radius, the Bohr radius, a 0. The energy of this state ($n = 1$), E_1 is -13.6 eV.

Ans. *(a)* The total energy associated with the two H-atoms in the ground state collide in elastically $= 2 \times (13.6 \text{ eV}) = 27.2$ eV.

The maximum amount by which their combined kinetic energy is reduced when any one of them goes into first excited state after the inelastic collision.

The total energy associated with the two H-atoms after the collision

$$= \left(\frac{13.6}{2^2}\right) + (13.6) = 17.0 \text{ eV}$$

Therefore, maximum loss of their combined kinetic energy

$$= 27.2 - 17.0 = 10.2 \text{ eV}$$

Q. 7 A set of atoms in an excited state decays

(a) in general to any of the states with lower energy
(b) into a lower state only when excited by an external electric field
(c) all together simultaneously into a lower state
(d) to emit photons only when they collide

K Thinking Process

The electron of atoms in excited states can fall back to a state of lower energy, emitting a photon in the process.

Ans. *(a)* A set of atoms in an excited state decays in general to any of the states with lower energy.

Multiple Choice Questions (More Than One Options)

Q. 8 An ionised H-molecule consists of an electron and two protons. The protons are separated by a small distance of the order of angstrom. In the ground state.

(a) the electron would not move in circular orbits

(b) the energy would be $(2)^4$ times that of a H-atom

(c) the electrons, orbit would go around the protons

(d) the molecule will soon decay in a proton and a H-atom

K **Thinking Process**

A hydrogen molecule contain two electrons and two protons whereas ionised H-molecule consists of an electron and two protons.

Ans. ***(a, c)***

The protons are separated by a small distance of the order of angstrom. In the ground state the electron would not move in circular orbits the electrons, orbit would go around the protons.

Q. 9 Consider aiming a beam of free electrons towards free protons. When they scatter, an electron and a proton cannot combine to produce a H-atom.

(a) Because of energy conservation

(b) Without simultaneously releasing energy in the form of radiation

(c) Because of momentum conservation

(d) Because of angular momentum conservation

Ans. ***(a, b)***

When beam of free electrons is aiming towards free protons. Then, they scatter but an electron and a proton cannot combine to produce a H-atom because of energy conservation and without simultaneously releasing energy in the form of radiation.

Q. 10 The Bohr model for the spectra of a H-atom

(a) will not be applicable to hydrogen in the molecular from

(b) will not be applicable as it is for a He-atom

(c) is valid only at room temperature

(d) predicts continuous as well as discrete spectral lines

K **Thinking Process**

Niel's Bohr proposed a model for hydrogenic (single electron) atoms in order to explain the line spectra emitted by atoms, as well as the stability of atoms.

Ans. ***(a, b)***

The Bohr model for the spectra of a H-atom will not be applicable to hydrogen in the molecular form. And also, it will not be applicable as it is for a He-atom.

Q. 11 The Balmer series for the H-atom can be observed

(a) if we measure the frequencies of light emitted when an excited atom falls to the ground state

(b) if we measure the frequencies of light emitted due to transitions between excited states and the first excited state

(c) in any transition in a H-atom

(d) as a sequence of frequencies with the higher frequencies getting closely packed

K **Thinking Process**

The various lines in the atomic spectra are produced when electrons jump from higher energy state to a lower energy state and photons are emitted. These spectral lines are called emission lines.

Ans. ***(b, d)***

Balmer series for the H-atom can be observed if we measure the frequencies of light emitted due to transitions between higher excited states and the first excited state and as a sequence of frequencies with the higher frequencies getting closely packed.

Q. 12 Let $E_n = \frac{-1\, me^4}{8\varepsilon_0^2 n^2 h^2}$ be the energy of the nth level of H-atom. If all the H-atoms are in the ground state and radiation of frequency $\frac{(E_2 - E_1)}{h}$ falls on it,

(a) it will not be absorbed at all

(b) some of atoms will move to the first excited state

(c) all atoms will be excited to the $n = 2$ state

(d) no atoms will make a transition to the $n = 3$ state

K **Thinking Process**

When an atom absorbs a photon that has precisely the same energy needed by the electron in a lower energy state to make transitions to a higher energy state, the process is called absorption.

Ans. ***(b, d)***

When all the H-atoms are in the ground state and radiation of frequency $\frac{(E_2 - E_1)}{h}$ falls on it, some of atoms will move to the first excited state and no atoms will make a transition to the $n = 3$ state.

Q. 13 The simple Bohr model is not applicable to He^4 atom because

(a) He^4 is an inert gas

(b) He^4 has neutrons in the nucleus

(c) He^4 has one more electron

(d) electrons are not subject to central forces

K **Thinking Process**

Neil's Bohr proposed a model for hydrogenic (single electron) atoms in order to explain the line spectra emitted by atoms, as well as the stability of atoms.

Ans. ***(c, d)***

The simple Bohr model is not applicable to He^4 atom because He^4 has one more electron and electrons are not subject to central forces.

Very Short Answer Type Questions

Q.14 The mass of a H-atom is less than the sum of the masses of a proton and electron. Why is this?

K **Thinking Process**

Einstein showed that mass is another form of energy and one can convert mass-energy into other forms of energy, say kinetic energy and vice-versa. Einstein gave the famous mass-energy equivalence relation $E = mc^2$ where the energy equivalent of mass m is related by the above equation and c is the velocity of light.

Ans. Since, the difference in mass of a nucleus and its constituents, ΔM, is called the mass defect and is given by

$$\Delta M = [Zm_p + (A - Z)m_n] - M$$

Also, the binding energy is given by $B =$ mass defect $(\Delta M) \times c^2$.

Thus, the mass of a H-atom is

$$m_p + m_e - \frac{B}{c^2}, \text{ where } B \approx 13.6 \text{ eV is the binding energy.}$$

Q. 15 Imagine removing one electron from He^4 and He^3. Their energy levels, as worked out on the basis of Bohr model will be very close. Explain why?

K **Thinking Process**

Neil's Bohr proposed a model for hydrogenic (single electron) atoms in order to explain the stability of atoms.

Ans. On removing one electron from He^4 and He^3, the energy levels, as worked out on the basis of Bohr model will be very close as both the nuclei are very heavy as compared to electron mass.Also after removing one electron from He^4 and He^3 atoms contain one electron and are hydrogen like atoms.

Q. 16 When an electron falls from a higher energy to a lower energy level, the difference in the energies appears in the form of electromagnetic radiation. Why cannot it be emitted as other forms of energy?

K **Thinking Process**

The accelerated electron produces electric as well as magnetic field hence electromagnetic energy.

Ans. The transition of an electron from a higher energy to a lower energy level can appears in the form of electromagnetic radiation because electrons interact only electromagnetically.

Q. 17 Would the Bohr formula for the H-atom remain unchanged if proton had a charge $(+4/3)e$ and electron a charge $(-3/4)e$, where $e = 1.6 \times 10^{-19}$ C. Give reasons for your answer.

K **Thinking Process**

The electrostatic force of attraction between positively charged nucleus and negatively charged electrons provides necessary centripetal force of revolution. Also, the magnitude of electrostatic force $F \propto q_1 q_2$.

Ans. If proton had a charge $(+4/3)e$ and electron a charge $(-3/4)e$, then the Bohr formula for the H-atom remain same, since the Bohr formula involves only the product of the charges which remain constant for given values of charges.

Q. 18 Consider two different hydrogen atoms. The electron in each atom is in an excited state. Is it possible for the electrons to have different energies but the same orbital angular momentum according to the Bohr model?

K **Thinking Process**

Bohr's postulate states that the electron revolves around the nucleus only in those orbits for which the angular momentum is some integral multiple of $\frac{h}{2\pi}$, where h is Planck's constant $(= 6.6 \cdot 10^{-34}$ J-s). Thus, the angular momentum (L) of the orbiting electron is quantised. i.e.,

$$L = \frac{nh}{2\pi}$$

Ans. According to Bohr model electrons having different energies belong to different levels having different values of n. So, their angular momenta will be different, as

$$L = \frac{nh}{2\pi} \text{ or } L \propto n$$

Short Answer Type Questions

Q. 19 Positronium is just like a H-atom with the proton replaced by the positively charged anti-particle of the electron (called the positron which is as massive as the electron). What would be the ground state energy of positronium?

K **Thinking Process**

The reduced mass m of two particle system of masses m_1 and m_2 is given by

$$\frac{1}{m} = \frac{1}{m_1} + \frac{1}{m_2}$$

Ans. The total energy of the electron in the stationary states of the hydrogen atom is given by

$$E_n = -\frac{me^4}{8n^2\varepsilon_0^2h^2}$$

where signs are as usual and the m that occurs in the Bohr formula is the reduced mass of electron and proton. Also, the total energy of the electron in the ground state of the hydrogen atom is –13.6 eV.For H-atom reduced mass m_e. Whereas for positronium, the reduced mass is

$$m \approx \frac{m_e}{2}$$

Hence, the total energy of the electron in the ground state of the positronium atom is

$$\frac{-13.6 \text{ eV}}{2} = -6.8 \text{ eV}$$

Q. 20 Assume that there is no repulsive force between the electrons in an atom but the force between positive and negative charges is given by Coulomb's law as usual. Under such circumstances, calculate the ground state energy of a He-atom.

K Thinking Process

The total energy of the electron in the nth stationary states of the hydrogen. Atom of hydrogen like atom of atomic number Z is given by

$$E_n = Z^2 \frac{-13.6 \text{ eV}}{n^2}$$

Ans. For a He -nucleus with charge 2e and electrons of charge $-e$, the energy level in ground state is

$$-E_n = Z^2 \frac{-13.6\text{eV}}{n^2} = 2^2 \frac{-13.6\text{eV}}{1^2} = -54.4\text{eV}$$

Thus, the ground state will have two electrons each of energy E and the total ground state energy would be $-(4\times 13.6)\text{eV} = -54.4\text{eV}$.

Q. 21 Using Bohr model, calculate the electric current created by the electron when the H-atom is in the ground state.

K Thinking Process

The electric current due to revolution of charge is given by $i = \frac{q}{T} = Q\left(\frac{1}{T}\right) = Q \times n$, *where n is frequency.*

Ans. The electron in Hydrogen atom in ground state revolves on a circular path whose radius is equal to the Bohr radius (a_n). Let the velocity of electron is v.

$\therefore$ Number of revolutions per unit time $= \frac{2\pi a_0}{v}$

The electric current is given by $i = \frac{q}{t}$, if q charge flows in time t. Here, $q = e$

The electric current is given by $i = \frac{2\pi a_0}{v} e$.

Q. 22 Show that the first few frequencies of light that is emitted when electrons fall to nth level from levels higher than n, arc approximate harmonics (*i.e.*, in the ratio 1 : 2 : 3 ...) when $n >> 1$.

K Thinking Process

The problem is based on the explanation of spectrum of hydrogen atom.

Ans. The frequency of any line in a series in the spectrum of hydrogen like atoms corresponding to the transition of electrons from $(n + p)$ level to nth level can be expressed as a difference of two terms;

$$v_{mn} = cRZ^2\left[\frac{1}{(n+p)^2} - \frac{1}{n^2}\right]$$

where, $m = n + p$, $(p = 1, 2, 3, \ldots)$ and R is Rydberg constant.

For $p << n$

$$v_{mn} = cRZ^2\left[\frac{1}{n^2}\left(1+\frac{p}{n}\right)^{-2} - \frac{1}{n^2}\right]$$

$$v_{mn} = cRZ^2\left[\frac{1}{n^2} - \frac{2p}{n^3} - \frac{1}{n^2}\right]$$

[By binomial theorem $(1+x)^n = 1 + nx$ if $|x| < 1$]

$$v_{mn} = cRZ^2\frac{2p}{n^3} \simeq \left(\frac{2cRZ^2}{n^3}\right)p$$

Thus, the first few frequencies of light that is emitted when electrons fall to the nth level from levels higher than n, are approximate harmonic (*i.e.*, in the ratio 1 : 2 : 3 ...) when $n >> 1$.

Q. 23 What is the minimum energy that must be given to a H-atom in ground state so that it can emit an H_γ line in Balmer series? If the angular momentum of the system is conserved, what would be the angular momentum of such H_γ photon?

Thinking Process

The third line in Balmer series in the spectrum of hydrogen atom is Hg.

Ans. H_γ in Balmer series corresponds to transition $n = 5$ to $n = 2$. So, the electron in ground state *i.e.*, from $n = 1$ must first be placed in state $n = 5$.

Energy required for the transition from $n = 2$ to $n = 5$ is given by

$$= E_1 - E_5 = 13.6 - 0.54 = 13.06 \text{ eV}$$

Since, angular momentum is conserved,

angular momentum coresponding to Hg photon = change in angular momentum of electron

$$= L_5 - L_2 = 5h - 2h = 3h = 3 \times 1.06 \times 10^{-34}$$
$$= 3.18 \times 10^{-34} \text{ kg-m}^2\text{/s}$$

Long Answer Type Questions

Q. 24 The first four spectral in the Lyman series of a H-atom are $\lambda = 1218$Å, 1028Å, 974.3Å and 951.4Å. If instead of Hydrogen, we consider deuterium, calculate the shift in the wavelength of these lines.

Thinking Process

The reduced mass m of two particle system of masses m_1 and m_2 is given by $\frac{1}{m} = \frac{1}{m_1} + \frac{1}{m_2}$.

Ans. The total energy of the electron in the stationary states of the hydrogen atom is given by

$$E_n = -\frac{me^4}{8n^2\varepsilon_0^2h^2}$$

where signs are as usual and the m that occurs in the Bohr formula is the reduced mass of electron and proton in hydrogen atom.

By Bohr's model,

$$h\nu_{if} = E_{n_i} - E_{n_f}$$

On simplifying,

$$\nu_{if} = \frac{me^4}{8\varepsilon_0^2 h^3}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

Since, $\lambda \propto \frac{1}{\mu}$

Thus, $$\lambda_{if} \propto \frac{1}{\mu} \quad \ldots\text{(i)}$$

where μ is the reduced mass. (here, μ is used in place of m)

Reduced mass for $H = \mu_H = \frac{m_e}{1+\frac{m_e}{M}}$; $m_e\left(1-\frac{m_e}{M}\right)$

Reduced mass for $D = \mu_D$; $m_e\left(1-\frac{m_e}{2M}\right)$

$$= m_e\left(1-\frac{m_e}{2M}\right)\left(1+\frac{m_e}{2M}\right)$$

If for hydrogen deuterium, the wavelength is $\frac{\lambda_H}{\lambda_D}$

$$\frac{\lambda_D}{\lambda_H} = \frac{\mu_H}{\lambda_D} \simeq \left(1+\frac{m_e}{2M}\right)^{-1} \simeq \left(1-\frac{1}{2\times 1840}\right) \quad \text{[From Eq. (i)]}$$

$$\lambda_D = \lambda_H \times (0.99973)$$

On substituting the values, we have

Thus, lines are 1217.7Å, 1027.7Å, 974.04Å, 951.143Å.

Q. 25 Deutrium was discovered in 1932 by Harold Urey by measuring the small change in wavelength for a particular transition in ^1H and ^2H. This is because, the wavelength of transition depend to a certain extent on the nuclear mass. If nuclear motion is taken into account, then the electrons and nucleus revolve around their common centre of mass.

Such a system is equivalent to a single particle with a reduced mass μ, revolving around the nucleus at a distance equal to the electron-nucleus separation. Here $\mu = m_e M/(m_e + M)$, where M is the nuclear mass and m_e is the electronic mass. Estimate the percentage difference in wavelength for the 1st line of the Lyman series in ^1H and ^2H. (mass of ^1H nucleus is 1.6725×10^{-27} kg, mass of ^2H nucleus is 3.3374×10^{-27} kg, Mass of electron $= 9.109 \times 10^{-31}$ kg.)

K **Thinking Process**

The percentage difference in wavelength is given by

$$100 \times \frac{\Delta\lambda}{\lambda_H} = \frac{\lambda_D - \lambda_H}{\lambda_H} \times 100, \quad \textit{where signs are as usual.}$$

Ans. The total energy of the electron in the *n*th states of the hydrogen like atom of atomic number Z is given by

$$E_n = -\frac{\mu Z^2 e^4}{8\varepsilon_0^2 h^2}\left(\frac{1}{n^2}\right)$$

where signs are as usual and the μ that occurs in the Bohr formula is the reduced mass of electron and proton .

Let μ_H be the reduced mass of hydrogen and μ_D that of Deutrium. Then, the frequency of the 1st Lyman line in hydrogen is $h\nu_H = \frac{\mu_H e^4}{8\varepsilon_0^2 h^2}\left(1-\frac{1}{4}\right) = \frac{\mu_H e^4}{8\varepsilon_0^2 h^2} \times \frac{3}{4}$.

Thus, the wavelength of the transition is $\lambda_H = \frac{3}{4}\frac{\mu_H e^4}{8\varepsilon_0^2 h^3 c}$. The wavelength of the transition for the same line in Deutrium is $\lambda_D = \frac{3}{4}\frac{\mu_D e^4}{8\varepsilon_0^2 h^3 c}$.

$\therefore$ $$\Delta\lambda = \lambda_D - \lambda_H$$

Hence, the percentage difference is

$$100 \times \frac{\Delta\lambda}{\lambda_H} = \frac{\lambda_D - \lambda_H}{\lambda_H} \times 100 = \frac{\mu_D - \mu_H}{\mu_H} \times 100$$

$$= \frac{\dfrac{m_e M_D}{(m_e + M_D)} - \dfrac{m_e M_H}{(m_e - M_H)}}{\dfrac{m_e M_H}{(m_e + M_H)}} \times 100$$

$$= \left[\left(\frac{m_e + M_H}{m_e + M_D}\right)\frac{M_D}{M_H} - 1\right] \times 100$$

Since, $m_e << M_H << M_D$

$$\frac{\Delta\lambda}{\lambda_H} \times 100 = \left[\frac{M_H}{M_D} \times \frac{M_D}{M_H}\left(\frac{1+\dfrac{m_e}{M_H}}{1+\dfrac{m_e}{M_D}}\right) - 1\right] \times 100$$

$$= \left[\left(1+\frac{m_e}{M_H}\right)\left(1+\frac{m_e}{M_D}\right)^{-1} - 1\right] \times 100 \simeq \left[1+\frac{m_e}{M_H} - \frac{m_e}{M_D} - 1\right] \times 100$$

[By binomial theorem, $(1+x)^n = 1 + nx$ is $|x| < 1$]

$$\approx m_e\left[\frac{1}{M_H} - \frac{1}{M_D}\right] \times 100$$

$$= 9.1 \times 10^{-31}\left[\frac{1}{1.6725 \times 10^{-27}} - \frac{1}{3.3374 \times 10^{-27}}\right] \times 100$$

$$= 9.1 \times 10^{-4}[0.5979 - 0.2996] \times 100$$

$$= 2.714 \times 10^{-2}\%$$

Q. 26 If a proton had a radius R and the charge was uniformly distributed, calculate using Bohr theory, the ground state energy of a H-atom when (i) $R = 0.1\text{Å}$ and (ii) $R = 10\text{Å}$.

Thinking Process

In this problem, expressions are to be derived in two cases namely for a point nucleus in H-atom and for an spherical nucleus of radius R.

Ans. The electrostatic force of attraction between positively charged nucleus and negatively charged electrons (Coulombian force) provides necessary centripetal force of revolution.

$$\frac{mv^2}{r_B} = -\frac{e^2}{r_B^2} \cdot \frac{1}{4\pi\varepsilon_0}$$

By Bohr's postulates in ground state, we have

$$mvr = h$$

On solving,

$$\therefore \quad m\frac{h^2}{m^2 r_B^2} \cdot \frac{1}{r_B} = +\left(\frac{e^2}{4\pi\varepsilon_0}\right)\frac{1}{r_B^2}$$

$$\therefore \quad \frac{h^2}{m} \cdot \frac{4\pi\varepsilon_0}{e^2} = r_B = 0.51\text{Å} \qquad \text{[This is Bohr's radius]}$$

The potential energy is given by

$$-\left(\frac{e^2}{4\pi r_0}\right) \cdot \frac{1}{r_B} = -27.2\text{eV}; \text{KE} = \frac{mv^2}{2}$$

$$= \frac{1}{2} m \cdot \frac{h^2}{m^2 r_B^2} = \frac{h}{2mr_B^2} = +13.6 \text{ eV}$$

Now, for an spherical nucleus of radius R,

If $R < r_B$, same result.

If $R >> r_B$ the electron moves inside the sphere with radius r'_B (r'_B = new Bohr radius).

Charge inside $\quad r_B'^4 = e\left(\frac{r_B'^3}{R^3}\right)$

$$\therefore \quad r'_B = \frac{h^2}{m}\left(\frac{4\pi\varepsilon_0}{e^2}\right)\frac{R^3}{r_B'^3}$$

$$r_B'^4 = (0.51\text{Å}) \cdot R^3 \qquad [R = 10\text{Å}]$$

$$= 510(\text{Å})^4$$

$$\therefore \quad r'_B \simeq (510)^{1/4}\text{Å} < R$$

$$\text{KE} = \frac{1}{2}mv^2 = \frac{m}{2} \cdot \frac{h}{m^2 r_B'^2} = \frac{h}{2m} \cdot \frac{1}{r_B'^2}$$

$$= \left(\frac{h^2}{2mr_B^2} \cdot \left(\frac{r_B^2}{r_B'^2}\right)\right) = (13.6 \text{ eV})\frac{(0.51)^2}{(510)^{1/2}} = \frac{3.54}{22.6} = 0.16 \text{ eV}$$

$$\text{PE} = +\left(\frac{e^2}{4\pi\varepsilon_0}\right) \cdot \left(\frac{r_B'^2 - 3R^2}{2R^3}\right)$$

$$= +\left(\frac{e^2}{4\pi\varepsilon_0} \cdot \frac{1}{r_B}\right) \cdot \left(\frac{r_B(r_B'^2 - 3R^2)}{R^3}\right) = +(27.2\text{eV})\left[\frac{0.51(\sqrt{510} - 300)}{1000}\right]$$

$$= +(27.2\text{eV}) \cdot \frac{-141}{1000} = -3.83 \text{ eV}$$

Q. 27 In the Auger process, an atom makes a transition to a lower state without emitting a photon. The excess energy is transferred to an outer electron which may be ejected by the atom (This is called an Auger, electron). Assuming the nucleus to be massive, calculate the kinetic energy of an $n = 4$ Auger electron emitted by Chromium by absorbing the energy from a $n = 2$ to $n = 1$ transition.

K **Thinking Process**

As the nucleus is massive, recoil momentum of the atom may be neglected and the entire energy of the transition may be considered transferred to the Auger electron. As there is a single valence electron in Cr, the energy states may be thought of as given by the Bohr model.

Ans. The energy of the nth state $E_n = -Z^2R\frac{1}{n^2}$ where R is the Rydberg constant and $Z = 24$.

The energy released in a transition from 2 to 1 is $\Delta E = Z^2R\left(1-\frac{1}{4}\right) = \frac{3}{4}Z^2R$.

The energy required to eject a $n = 4$ electron is $E_4 = Z^2R\frac{1}{16}$.

Thus, the kinetic energy of the Auger electron is

$$\text{KE} = Z^2R\left(\frac{3}{4}-\frac{1}{16}\right) = \frac{1}{16}Z^2R$$

$$= \frac{11}{16} \times 24 \times 24 \times 13.6\text{eV}$$

$$= 5385.6\text{eV}$$

Q. 28 The inverse square law in electrostatic is $|\mathbf{F}| = \frac{e^2}{(4\pi\varepsilon_0)r^2}$ for the force between an electron and a proton. The $\left(\frac{1}{r}\right)$ dependence of $|\mathbf{F}|$ can be understood in quantum theory as being due to the fact that the particle of light (photon) is massless. If photons had a mass m_p, force would be modified to $|\mathbf{F}| = \frac{e^2}{(4\pi\varepsilon_0)r^2}\left[\frac{1}{r^2}+\frac{\lambda}{r}\right]\cdot\exp(-\lambda r)$ where $\lambda = \frac{m_p c}{\hbar}$ and $\hbar = \frac{h}{2\pi}$. Estimate the change in the ground state energy of a H-atom if m_p were 10^{-6} times the mass of an electron.

Ans. For $m_p = 10^{-6}$ times, the mass of an electron, the energy associated with it is given by

$$m_pc^2 = 10^{-6} \times \text{electron mass} \times c^2$$

$$\approx 10^{-6} \times 0.5 \text{ MeV}$$

$$\approx 10^{-6} \times 0.5 \times 1.6 \times 10^{-13}$$

$$\approx 0.8 \times 10^{-19}\text{J}$$

The wavelength associated with is given by

$$\frac{\hbar}{m_p c} = \frac{\hbar c}{m_p c^2} = \frac{10^{-34} \times 3 \times 10^8}{0.8 \times 10^{-19}}$$

$$\approx 4 \times 10^{-7} \text{m} >> \text{Bohr radius}$$

$$|\mathbf{F}| = \frac{e^2}{4\pi\varepsilon_0}\left[\frac{1}{r^2} + \frac{\lambda}{r}\right]\exp(-\lambda r)$$

where, $$\lambda^{-1} = \frac{\hbar}{m_p c} \approx 4 \times 10^{-7} \text{m} >> r_B$$

$\therefore \quad \lambda << \frac{1}{r_B}$ *i.e.,* $\lambda r_B << 1$

$$U(r) = -\frac{e^2}{4\pi\varepsilon_0} \cdot \frac{\exp(-\lambda r)}{r}$$

$$mvr = \hbar \quad \therefore \quad v = \frac{\hbar}{mr}$$

Also, $$\frac{mv^2}{r} = \approx \left(\frac{e^2}{4\pi\varepsilon_0}\right)\left[\frac{1}{r^2} + \frac{\lambda}{r}\right]$$

$\therefore$ $$\frac{\hbar^2}{mr^3} = \left(\frac{e^2}{4\pi\varepsilon_0}\right)\left[\frac{1}{r^2} + \frac{\lambda}{r}\right]$$

$\therefore$ $$\frac{\hbar^2}{m} = \left(\frac{e^2}{4\pi\varepsilon_o}\right)[r + \pi r^2$$

If $\lambda = 0$; $$r = r_B = \frac{\hbar}{m} \cdot \frac{4\pi\varepsilon_0}{e^2}$$

$$\frac{\hbar^2}{m} = \frac{e^2}{4\pi\varepsilon_0} \cdot r_B$$

Since, $\lambda^{-1} >> r_B$, put $r = r_B + \delta$

$\therefore$ $$r_B = r_B + \delta + \lambda(r_B^2 + \delta^2 + 2\delta r_B); \text{ neglect } \delta^2$$

or $$0 = \lambda r_B^2 + \delta(1 + 2\lambda r_B)$$

$$\delta = \frac{-\lambda r_B^2}{1 + 2\lambda r_B} \approx \lambda r_B^2(1 - 2\lambda r_B) = -\lambda r_B^2$$

Since, $\lambda r_B << 1$

$\therefore$ $$V(r) = -\frac{e^2}{4\pi\varepsilon_0} \cdot \frac{\exp(-\lambda\delta - \lambda r_B)}{r_B + \delta}$$

$\therefore$ $$V(r) = -\frac{e^2}{4\pi\varepsilon_0}\frac{1}{r_B}\left[\left(1 - \frac{\delta}{r_B}\right) \cdot (1 - \lambda r_B)\right]$$

$$\cong (-27.2 \text{eV}) \text{ remains unchanged}$$

$$\text{KE} = -\frac{1}{2}mv^2 = \frac{1}{2}m \cdot \frac{h^2}{mr^2} = \frac{h^2}{2(r_B + \delta)^2} = \frac{h^2}{2r_B^2}\left(1 - \frac{2\delta}{r_B}\right)$$

$$= (13.6 \text{eV})[1 + 2\lambda r_B]$$

Total energy $$= -\frac{e^2}{4\pi\varepsilon_0 r_B} + \frac{h^2}{2r_B^2}[1 + 2\lambda r_B]$$

$$= -27.2 + 13.6[1 + 2\lambda r_B] \text{eV}$$

Change in energy $$= 13.6 \times 2\lambda r_B \text{eV} = 27.2\lambda r_B \text{eV}$$

Q. 29 The Bohr model for the H-atom relies on the Coulomb's law of electrostatics. Coulomb's law has not directly been verified for very short distances of the order of angstroms. Supposing Coulomb's law between two opposite charge $+q_1, -q_2$ is modified to

$$|\mathbf{F}| = \frac{q_1 q_2}{(4\pi\varepsilon_0)} \frac{1}{r^2}, r \geq R_0$$

$$= \frac{q_1 q_2}{4\pi\varepsilon_0} \frac{1}{R_0^2} \left(\frac{R_0}{r}\right)^{\varepsilon}, r \leq R_0$$

Calculate in such a case, the ground state energy of a H-atom, if $E = 0.1$, $R_0 = 1$Å.

K **Thinking Process**

The question offers hypothetical situation in dealing with the total energy of the electron of hydrogen atom.

Ans. Considering the case, when $r \leq R_0 = 1$ Å

$$\text{Let } \varepsilon = 2 + \delta$$

$$F = \frac{q_1 q_2}{4\pi\varepsilon_0} \cdot \frac{R_0^{\delta}}{r^{2+\delta}}$$

where, $$\frac{q_1 q_2}{4\pi_0\varepsilon_0} = (1.6 \times 10^{-19})^2 \times 9 \times 10^9$$

$$= 23.04 \times 10^{-29} \text{N m}^2$$

The electrostatic force of attraction between positively charged nucleus and negatively charged electrons (Coulombian force) provides necessary centripetal force.

$$= \frac{mv^2}{r} \quad \text{or} \quad v^2 = \frac{\wedge R_0^{\delta}}{mr^{1+\delta}} \quad \ldots \text{(i)}$$

$$mvr = nh \cdot r = \frac{n\hbar}{mv} = \frac{nh}{m}\left[\frac{m}{\wedge R_0^{\delta}}\right]^{1/2} r^{1/2 + \delta/2}$$

[Applying Bohr's second postulates]

Solving this for r, we get $$r_n = \left[\frac{n^2\hbar^2}{m \wedge R_0^{\delta}}\right]^{\frac{1}{1-\delta}}$$

where, r_n is radius of nth orbit of electron.
For $n = 1$ and substituting the values of constant, we get

$$r_1 = \left[\frac{\hbar^2}{m \wedge R_0^{\delta}}\right]^{\frac{1}{1-\delta}}$$

$$r_1 = \left[\frac{1.05^2 \times 10^{-68}}{9.1 \times 10^{-31} \times 2.3 \times 10^{-28} \times 10^{+19}}\right]^{\frac{1}{2.9}}$$

$$= 8 \times 10^{-11}$$

$$= 0.08 \text{ nm} \quad (< 0.1 \text{ nm})$$

This is the radius of orbit of electron in ground state of hydrogen atom.

$$v_n = \frac{n\hbar}{mr_n} = n\hbar\left(\frac{m \wedge R_0^{\delta}}{n^2\hbar^2}\right)^{\frac{1}{1-\delta}}.$$

$$\text{For } n = 1,\ v_1 - \frac{\hbar}{mr_1} = 1.44 \times 10^6 \text{ m/s}$$

[This is the speed of electron in ground state]

$$\text{KE} = \frac{1}{2}mv_1^2 - 9.43 \times 10^{-19} \text{ J} = 5.9\text{eV}$$

[This is the KE of electron in ground state]

PE till $R_0 = -\frac{\wedge}{R_0}$ [This is the PE of electron in ground state at $r = R_0$]

$$\text{PE from } R_0 \text{ to } r = + \wedge R_0^{\delta} \int_{R_0}^{r} \frac{dr}{r^{2+\delta}} = + \frac{\wedge R_0^{\delta}}{-1-\delta}\left[\frac{1}{r^{1+\delta}}\right]_{R_0}^{r}$$

[This is the PE of electron in ground state at R_0 to r]

$$= -\frac{\wedge R_0^{\delta}}{1+\delta}\left[\frac{1}{r^{1+\delta}} - \frac{1}{R_0^{1+\delta}}\right] = -\frac{\wedge}{1+\delta}\left[\frac{R_0^{\delta}}{r^{1+\delta}} - \frac{1}{R_0}\right]$$

$$\text{PE} = -\frac{\wedge}{1+\delta}\left[\frac{R_0^{\delta}}{r^{1+\delta}} - \frac{1}{R_0} + \frac{1+\delta}{R_0}\right]$$

$$\text{PE} = -\frac{\wedge}{-0.9}\left[\frac{R_0^{-1.9}}{r^{-0.9}} - \frac{1.9}{R_0}\right]$$

$$= \frac{2.3}{0.9} \times 10^{-18}[(0.8)^{0.9} - 1.9]\text{J} = -17.3 \text{ eV}$$

Total energy is $(-17.3 + 5.9) = -11.4$ eV

This is the required TE of electron in ground state.